If $\cos\phi=\infty$, $\tan\phi=\pm i$.

If $\sin 15\phi=0$, $\phi=\frac{n\pi}{15}$ $(n=0, 1, 2, \ldots, 14)$.

$\therefore x=\pm i, 0, \tan\frac{\pi}{15}, \tan\frac{2\pi}{15}, \ldots, \tan\frac{14\pi}{15}$,

$$=\pm i, 0, \pm\frac{\sqrt{[10+2\sqrt{(5)}]}-\sqrt{(15)}+\sqrt{(3)}}{\sqrt{[30+6\sqrt{(5)}]}+\sqrt{(5)}-1}, \pm\frac{\sqrt{(15)}+\sqrt{(3)}-\sqrt{[10-2\sqrt{(5)}]}}{\sqrt{[30-6\sqrt{(5)}]}+\sqrt{(5)}+1},$$

$$\pm\frac{\sqrt{[10-2\sqrt{(5)}]}}{\sqrt{(5)}+1}, \pm\frac{\sqrt{[10+2\sqrt{(5)}]}+\sqrt{(15)}-\sqrt{(3)}}{\sqrt{[30+6\sqrt{(5)}]}-\sqrt{(5)}+1}, \pm\sqrt{(3)},$$

$$\pm\frac{\sqrt{[10+2\sqrt{(5)}]}}{\sqrt{(5)}-1}, \pm\frac{\sqrt{(15)}+\sqrt{(3)}+\sqrt{[10-2\sqrt{(5)}]}}{\sqrt{[30-6\sqrt{(5)}]}-\sqrt{(5)}-1}.$$

(These values are taken from Hobson's *Plane Trigonometry.*)

Professor J. Scheffer gave a partial solution, following the method in the above solution. Mr. Spuner sent in a similar solution, with results given in decimals instead of radicals.

347. Proposed by GUSTAVE JACOBSON, A. M., Public Accountant, Chicago, Ill.

A corporation needing some additional capital for a short term of years, issues \$300,000 of debenture bonds carrying 6% interest, and payable 1/5 each year for 5 years. Coupons are attached to the bonds maturing every six months; the bonds are sold at 90 flat. What average rate of interest does the company pay for the money, including interest on interest?

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Let $2x$=the rate of interest paid per annum, payable semi-annually; then x=the rate of semi-annual interest, compound. Only \$270,000 was used by the corporation, which was repaid, interest in 10 payments and principal in 5 payments; and they alternate, the first each six months and the second at the end of each year as follows: (1), \$9,000; (2), \$69,000, (3), \$7,200; (4), \$67,200; (5), \$5,400; (6), \$65,400; (7), \$3,600; (8), \$63,600; (9), \$1,800, and (10), \$61,800.

The problem may now be treated, algebraically, as a question in partial payments under the U. S. Court Rule; and we have, after taking out the factor 600, arranging, dropping the dollar sign as follows:

$$450(1+x)^{10}-[15(1+x)^9+115(1+x)^8+12(1+x)^7+112(1+x)^6$$
$$+9(1+x)^5+109(1+x)^4+6(1+x)^3+106(1+x)^2+3(1+x)]=103.$$

Observe that as arranged the first member has 2 terms, and their difference

$=103$, As the debenture was discounted, the rate realized on the investment must be greater than 3% as a semi-annual rate. We try 5% and find it too small; we then try 5.0275%, with $(1+x)=1.050275$, and find the first member$=102.9337$, which is too small; we then try $(1+x)=1.050285$ and obtain the value 102.9531, still too small. By double position we have a corrected value and $(1+x)=1.050309$ and obtain 103.0304, which is now too large. By double positon we have $(1+x)=1.050291$, and obtain 102.9937, again too small. By double position again we have $(1+x)=1.0502941$ and obtain 103.0033, now too large. We now again use double position and obtain, $(1+x)=1.05029269$, this will give a value which will differ by less than 1% of a mill. We now have for a good approximation a semi-annual rate, 5.029269%; or the rate per annum payable semi-annually, 10.058538%.

Also solved by S. A. Corey who got for a result 10.28 per cent.; A. H. Holmes, whose result was 10.07 per cent., and the proposer, whose result was 10.0532 per cent. These contributors used practically the same method of solution. Mr. Spuner also sent in an excellent solution of 345, which reached us too late for credit in the January number.

CALCULUS.

298. Proposed by C. N. SCHMALL, New York City.

Prove, by calculus, that if two regular polygons have equal perimeters, that which has the greater number of sides has the greater area.

Solution by E. L. SHERWOOD, Shady Side Academy, Pittsburg, Penn., and the PROPOSER.

In any regular polygon, let $2p$=the perimeter, n=number of sides.

$\therefore \dfrac{2p}{n}$=each side, and $\dfrac{p}{n\tan(\pi/n)}$=the apothem.

$$\frac{p^2}{n\tan(\pi/n)}=\frac{p^2}{n}\cot\frac{\pi}{n}=\text{area}\ldots(1).$$

Put $u=n\tan\dfrac{\pi}{n}\ldots(2)$; hence the area will be a maximum when u is a minimum.

From (2) we have

$$\frac{du}{dn}=\tan\frac{\pi}{n}-\frac{\pi}{n}\sec^2\frac{\pi}{n}=\tan\phi-\phi\sec^2\phi \text{ (where } \frac{\pi}{n}=\phi)=\sin\phi\sec\phi-\phi\sec^2\phi$$

$$=\tfrac{1}{2}(2\sin\phi\sec\phi-2\phi\sec^2\phi)=\tfrac{1}{2}\sec^2\phi\left(\frac{2\sin\phi}{\sec\phi}-2\phi\right)=\tfrac{1}{2}\sec^2\phi(\sin2\phi-2\phi),$$

$$=\tfrac{1}{2}\sec^2\frac{\pi}{n}\left(\sin\frac{2\pi}{n}-\frac{2\pi}{n}\right).$$